# Mathematics Test Three—Scoring Key

| | | Subscore Area | | |
|---|---|---|---|---|
| | Key | EA | AG | GT |
| 1. | B | ___ | | |
| 2. | K | | | ___ |
| 3. | D | ___ | | |
| 4. | F | ___ | | |
| 5. | E | ___ | | |
| 6. | J | ___ | | |
| 7. | A | | | ___ |
| 8. | G | ___ | | |
| 9. | D | ___ | | |
| 10. | F | ___ | | |
| 11. | E | ___ | | |
| 12. | K | | ___ | |
| 13. | E | | | ___ |
| 14. | G | | ___ | |
| 15. | E | ___ | | |
| 16. | F | ___ | | |
| 17. | E | ___ | | |
| 18. | K | ___ | | |
| 19. | C | ___ | | |
| 20. | H | | ___ | |
| 21. | B | ___ | | |
| 22. | G | | | ___ |
| 23. | D | ___ | | |
| 24. | J | ___ | | |
| 25. | B | ___ | | |
| 26. | J | | ___ | |
| 27. | C | | ___ | |
| 28. | F | | ___ | |
| 29. | C | | ___ | |
| 30. | F | | | ___ |

| | | Subscore Area | | |
|---|---|---|---|---|
| | Key | EA | AG | GT |
| 31. | B | | ___ | |
| 32. | H | | | ___ |
| 33. | D | | | ___ |
| 34. | F | | ___ | |
| 35. | B | | | ___ |
| 36. | J | | ___ | |
| 37. | D | ___ | | |
| 38. | K | | ___ | |
| 39. | C | | ___ | |
| 40. | H | | | ___ |
| 41. | C | ___ | | |
| 42. | J | ___ | | |
| 43. | A | | | ___ |
| 44. | G | | | ___ |
| 45. | E | | | ___ |
| 46. | J | | ___ | |
| 47. | C | | ___ | |
| 48. | G | | | ___ |
| 49. | E | | | ___ |
| 50. | J | | | ___ |
| 51. | B | | | ___ |
| 52. | H | | ___ | |
| 53. | B | | ___ | |
| 54. | F | | ___ | |
| 55. | A | | | ___ |
| 56. | H | ___ | | |
| 57. | A | ___ | | |
| 58. | K | | | ___ |
| 59. | A | | ___ | |
| 60. | H | ___ | | |

Number Correct (Raw Score) for:

| | |
|---|---|
| Pre-Algebra/Elementary Algebra (EA) Subscore | _____ (24) |
| Intermediate Algebra/Coordinate Geometry (AG) Subscore | _____ (18) |
| Plane Geometry/Trigonometry (GT) Subscore | _____ (18) |
| Total Number Correct for Math Test (EA + AG + GT) | ______ (60) |

Use the conversion chart on page 142 to convert the total number correct to the ACT Scoring Scale of 1 to 36.

 

# Mathematics Test Three—Answer Explanations

1. $10^2 = 100$, and $10^9 = 1{,}000{,}000{,}000$. Adding these values produces the sum of 1,000,000,100. The answer is B.

2. If x equals 45°, then z and u also = 45°. The measure of angle t equals 180° - 45° = 135°. The answer is K.

3. Because rate is equal to distance divided by time, divide 450 by 8.25: 450/8.25 = 54.5 to the nearest tenth of a mile. Thus, the answer is D.

4. $2x^2 + 10x - 12$ may be factored into $(x + 6)(2x - 2)$. Thus, the answer is F.

5. There are 8 green balls in the container containing 12 balls total. Hence, there is an 8 in 12 chance that a green ball will be the first drawn from the container. Thus, the probability is 8/12 or 2/3. The answer is E.

6. First, divide the total price for both pairs of shoes into three parts by dividing $52.50 by 3: $52.50/3 = $17.50. This is the price of the running shoes. To obtain the price of the sandals, multiply $17.50 times 2 = $35.00. The answer is J.

7. The measure of angle XAZ is equal to the measure of angle YAW. Because line RA bisects angle YAW, the measure of angle S equals 1/2 of 40°. The answer is A.

8. $\sqrt{2}$ is an irrational number because it cannot be written as a fraction, where both the numerator and denominator are integers. The answer is G.

9. Find the total number of tests: 22 + 27 + 21 = 70. Then, find the number of tests that Katie has graded in the past 3 hours: 16 + 3 + 19 = 48. Next, create a fraction where the numerator is the number of tests graded and the denominator is the total number of tests: 48/70. Finally, reduce the fraction: 48/70 = 24/35. The answer is D.

10. Begin by setting up an equation with the given information: $(0.20)y = 30$. Then solve for y by dividing both sides of the equation by 0.20. The answer is F.

11. Substitute the given values into the equation and solve: $S = cd + p[q(2r - 1) + 2r] = (10)(13) + 5[5(2(4) - 1) + 2(4)]$: $S = 235$. The answer is E.

12. Rearrange the equation into slope-intercept form, which is $y = mx + b$, where m is the slope of the line and b is the y-intercept. This can be done by subtracting 5x from each side and then dividing each side of the equation by 2: $y = -5/2(x) + 5$. The answer is K.

13. To find the number of tiles needed, divide 15 by 1.5: 15/1.5 = 10. Also, divide 9 by 1.5: 9/1.5 = 6. Then, multiply 10 by 6: 10(6) = 60. The answer is E.

14. Recall that the distance between two points $(x_1, y_1)$ and $(x_2, y_2)$ is equal to $\sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$. Substitute the given information into the distance formula: $\sqrt{(3 - 7)^2 + (3 - 2)^2} = \sqrt{(16 + 1)} = \sqrt{17}$. The answer is G.

15. In order to find the number of pies baked over the 5-day period, begin by writing an equation that represents the given information: total pies baked = $2(2z) + 3(3z - 2)$. Then, expand the information within parentheses: $4z + 9z - 6$. Finally, solve by combining common terms: $13z - 6$. The answer is E.

16. First, expand the given formula: $3z - 7y + 2(3x - z) + 4y = 3z - 7y + 6x - 2z + 4y$. Next, combine similar terms: $6x + (4y - 7y) + (3z - 2z) = 6x - 3y + z$. The answer is F.

17. The smallest number of which 3, 5, and 10 are factors is 30. Thus, the staff will clean all of the cages every 30th day. The answer is E.

18. Start by writing 4% as a decimal: 4% = 0.04. Then multiply 0.04 times 0.1 = 0.004. The answer is K.

19. Substitute the given values of p and q into the equation and solve: $p^3 - 2p^2q + 3pq^3 - q^3 = 6^3 - 2(6)^2(-3) + 3(6)(-3)^3 - (-3)^3 = 216 - (-216) + (-486) - (-27) = 216 + 216 - 486 + 27 = -27$. The answer is C.

 

20. Rewrite the equation of the line in slope-intercept form, $y = mx + b$, where m is the slope of the line and b is the y-intercept. This can be done by adding $(1/4)x$ to each side of the equation: $y = (1/4)x + 2/5$. Thus it can be seen that the slope of the line is 1/4. The answer is H.

21. Begin by finding a common denominator for the two fractions and then add. In this case, the lowest common denominator is equal to 12: $(x/3) + (x/4) = 1$: $(4x/12) + (3x/12) = 12/12$. Next, multiply both sides of the equation by 12: $4x + 3x = 12$: $7x = 12$. Then solve for x by dividing each side by 7: $x = 12/7$. The answer is B.

22. The tangent of X is equal to the length of the side opposite of X divided by the length of the side adjacent to X: x/z. The answer is G.

23. Expand the factor: $(2\sqrt{5} - 3)(\sqrt{5} + 4) = 10 - 3\sqrt{5} + 8\sqrt{5} - 12$. Then, combine similar terms: $10 + 5\sqrt{5} - 12 = -2 + 5\sqrt{5}$. The answer is D.

24. In order to find the area of the football field, multiply the length of the field by the width of the field: $(2x - 3)(3x^2 + 5) = 6x^3 - 9x^2 + 10x - 15$. The answer is J.

25. Multiply the cost of gas per gallon (0.97) by the total number of miles that Helen will be traveling (x): 0.97x. Then, divide by the number of miles per gallon Helen is planning to drive with her car (0.97x)/r. The answer is B.

26. From the graph, it can be seen that every year the store's profit increases by an annual amount of $100. Because the store's annual income was $1200 in 1998 and 2000 is two years ahead, it can be projected that the annual profit in 2000 will be equal to $1200 + 2($100) = $1400. The answer is J.

27. In the given equation, $(-5)^{-2} = 1/25$, and $(-1)^{-2} = 1$. Hence: $1/25 + 1 = 26/25$. The answer is C.

28. Remove the absolute value signs and make the left side of the inequality negative: $-(-4 - x) \geq 6$. Next, expand the left side of the inequality: $4 + x \geq 6$. Subtract 4 from both sides: $4 + x - 4 \geq 6 - 4$. Thus $x \geq 2$ and the smallest possible value for x is 2. The answer is F.

29. First, solve for x in the given equations: $x\sqrt{2} = 6$: $x = 6/(\sqrt{2}) = 3\sqrt{2}$. Thus, $x^2 = (3\sqrt{2})^2 = 18$. The answer is C.

30. Because a circle's circumference is equal to $\pi d$, and the given circumference of each of the smaller circles is $8\pi$, we know that the diameter of each of the small circles is equal to 8 centimeters. Thus, the diameter of the largest circle equals: 4(8) = 32. It also follows that the circumference of the largest circle equals $\pi d = 32\pi$. The answer is F.

31. Multiply like terms: $c^2$ times $c^2 = c^4$. Then $c^4 / c^2 = c^2$. Multiply $d^5$ times $d^2 = d^7$. Then $d^7 / d^6 = d$. Finally, 15 times -21 divided by 7 = -45. The answer is B.

32. From the figure it can be seen that BPC measures 40° and CBP measures 90°. To find the measure of BCP, subtract 90° and 40° from 180°: 180° - 90° - 40° = 50°. Thus, the measure of ACN equals 180° - 50° = 130°. The answer is H.

33. Multiply 8(2) = 16 in order to calculate the area of the rectangular portion. In order to calculate the area of the triangular portion, use the equation 1/2bh, where b is equal to the length of the triangle's base and h is equal to the height of the triangle, perpendicular to the chosen base: 1/2(4)(4) = 8. Then, add the areas of the rectangle and the triangle in order to find the total area: 16 + 8 = 24. The answer is D.

34. Minimize $(x + 3)^2$ by setting $(x + 3)$ equal to zero and solving for x: $(x + 3) = 0$: $x = -3$. Thus, it is when $x = -3$ that $12 - (x + 3)^2$ has its maximum value. The answer is F.

35. The area of the circle can be found using the equation $\pi r2$, where r is the length of the circle's radius. In this case, the radius of the circle is equal to 3: $\pi(3)^2 = 9\pi$. In order to calculate the area of the shaded region, subtract the area of the circle from the given area of the triangle (50): $50 - 9\pi$. The answer is B.

36. The given point (2, -2) is 2 units directly above the center of the circle. Hence, the opposite endpoint of the diameter is located 2 points directly below the center. The coordinates of this point can be found by subtracting 2 units from the y-value of the circle's center: (2, -6). The answer is J.

37. By definition, the denominator of a fraction (a - b) cannot be equal to zero. However, in order for the fraction to be equal to zero, the numerator (a + b) must have a zero value. Thus, the answer is D.

 

38. First, remember that the distance between two points $(x_1, y_1)$ and $(x_2, y_2)$ is equal to $\sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$. Substitute the given information into the distance formula: $\sqrt{(8 - 5)^2 + (12 - 2)^2} = \sqrt{(9 + 100)} = \sqrt{109}$. The answer is K.

39. For option A, the slope is 4: for option B, the slope is 5: for option C, the slope is 6: and for option D and E, the slope is 2. The answer is C.

40. The cotangent of the angle is equal to the cosine of the angle divided by the sine of the angle: 0.6/0.8 = 3/4. The answer is H.

41. Begin by substituting values for v into the answer options, such as v = 1 and v = 2. For any integer value of v, $2v^2$ will always yield an even integer. Hence, the answer is C.

42. Expand $(p-q)^2 = 16$: $p^2 - 2pq + q^2 = 16$. Substitute pq = 5 to get $p^2 - 10 + q^2 = 16$. Then $p^2 + q^2 = 26$. The answer is J.

43. Because QUT and RST both measure 135°, measures of VUT and VST can be calculated: 135° - 90° = 45°. Hence, UST is a 45° -45° -90° triangle, and the area of this portion is 1/2(base)(height) = 1/2(10)(5) = 25. The area of the square portion is 10(10) = 100. The area of the pentagon is the sum of the two areas: 100 + 25 = 125. The answer is A.

44. The measure of QPM equals the measure of LPR. Because NP bisects LPR, the measure of angle X will equal 1/2 of 130°, which is 65°. The answer is G.

45. Because of the angle measures given, ABC measure 86° and BAH measures 180° - 122° = 58°. Thus, BHA measures 180° - (86° + 58°) = 36°. Finally, CHE measures 36°. The answer is E.

46. In order to find the equation of the line, begin by finding the slope, which is equal to $(y_1 - y_2)/(x_1 - x_2)$. In this case, the slope is equal to (6-0)/(0-3) = -2. Next, find the y-intercept or the y-value where the line crosses the y axis: 6. Finally, place the calculated slope and y-intercept into slope intercept form: y = mx + b, where m is the slope of the line and b is the y-intercept. Thus, the equation of the line is y = -2x + 6. The answer is J.

47. First, write the equations in slope-intercept form: y = mx + b, where m is the slope of the line and b is the y-intercept: y = -3x + 7/2 and y = -3x + 4. Thus, it can be seen that the slope of these lines is -3. In order to find the perpendicular slope, take the reciprocal of the slope and negate it. Thus, the slope of the line perpendicular to those with a slope of -3 is 1/3. The answer is C.

48. Imagine that a line has been drawn, connecting points Q and T. This line is assumed to be the hypotenuse of a right triangle. The base of this triangle equals 7 + 2 = 9. The height of the triangle is 3. Next, use the Pythagorean Theorem to solve for the length of the hypotenuse: $a^2 + b^2 = c^2$: $9^2 + 3^2 = c^2$: $81 + 9 = c^2$: $c = \sqrt{90}$ or $c = 3\sqrt{10}$. The answer is G.

49. When the value is defined as $3\pi / 2 < b < 2\pi$ and sin b = -1/2, cos b must equal $-\sqrt{3}/2$ by definition. The answer is E.

50. Because the three points lie in a plane, it is possible for the three points to fall along a line. Supposing that these points fall along a straight line in the order ABC, points A and C would be 10 feet apart from one another (8 + 2 = 10). Supposing that these points fall along a straight line in the order ACB, points A and C would be 6 feet apart from one another (8 - 2 = 6). Thus, the minimum distance is 6 and the maximum distance is 10. All other arrangements of the three points produce distances between A and C that fall between these two extremes. The answer is J.

51. In order to find the area of the parallelogram, multiply the length of the base (3) by the height of the parallelogram perpendicular to the base (5): 3 x 5 = 15. The answer is B.

52. The equation of a circle is $(x - h)^2 + (y - k)^2 = r^2$, where (h, k) is the center of the circle, and r is the radius of the circle. Thus, in this case, the center of the circle is (-2, 3), and the radius is 2. Therefore, the greatest y-coordinate on the circle is two units directly above the y-coordinate at the circle's center: 3 + 2 = 5. The answer is H.

53. Remove the absolute value signs and solve the inequality: 3x + 2 < 4. First, subtract 2 from both sides: 3x < 2. Then, divide both sides by 3: x < 2/3. Next, refer back to the original inequality; remove the absolute value signs, and make the left side of the inequality negative: -(3x + 2) < 4. Expand: -3x - 2 < 4. Then, add 2 to both sides: -3x < 6. Next, divide each side by -3, remembering to change the direction of the inequality sign: x > -2. Hence, -2 < x < 2/3. The answer is B.

 

54. First, find the equation of the line. The slope is equal to the change in y divided by the change in x: $(y_1 - y_2)/(x_1 - x_2) = 2/1 = 2$. The y-intercept is equal to 2, because this is the y-value when the line crosses the y-axis. Thus, the equation of the line is $y = 2x + 2$. Because the shaded portion of the graph is below the line, the equal sign is changed: $y \leq 2x + 2$. The answer is F.

55. The trigonometric function that relates an angle's opposite and adjacent sides is tangent, where the tangent of an angle is equal to the length of the side opposite the angle divided by the length of the side adjacent to the angle. In this case, let the height of the tower be equal to x: $\tan 53° = x/50$. Next, solve for x by multiplying each side of the equation by 50: $x = 50 \tan 53°$. The answer is A.

56. The smallest number greater that 1 that, when divided by 2, 3, 5, or 6, leaves a remainder of 1 is 31: 31/2 = 15 with a remainder of 1; 31/3 = 10 with a remainder of 1; 31/5 = 6 with a remainder of 1; 31/6 = 5 with a remainder of 1. Thus, the answer is H.

57. Use the answer options in order to substitute the possible values for b into the given equation. By doing so, it can be seen that the appropriate value for b is 4: $15x^2 + 23x + 4 = (5x + 1)(3x + 4)$. Thus, the answer is A.

58. Begin by finding the surface area of one side of the large cube. This can be accomplished by dividing the total surface area (3750 square inches) by the number of sides (6): 3750/6 = 625. Thus, it can be seen that the length of one side of the cube is equal to $\sqrt{625} = 25$. Next find the surface area of one side of the small cube by dividing total surface area by the number of sides: 150/6 = 25. Hence, the length of the smaller square's side is equal to $\sqrt{25} = 5$. The next step is to calculate the volume of each of the cubes, remembering that volume of a cube is equal to the length of the cube's side, cubed. For the large cube, volume = $25^3$ =15,625 and for the small cube, volume = $5^3$ = 125. Lastly, in order to find the number of small cubes that can be created from the large cube, divide the large cube's volume by that of the small cube's volume: 15,625/125 = 125. The answer is K.

59. Because we are looking for negative values of x that solve the equation, remove the absolute value signs and negate the right side of the inequality: $-(2x + 3) \geq 10$. Then, solve the equation in terms of x: $-2x - 3 \geq 10$. Then $-2x \geq 13$.
Finally, $x \leq -13/2$. The answer is A.

60. Examine the answer options to see if they fit the paradigm described. By doing so, it can be seen that the number 90 is the smallest integer that is divisible by 18 and 10. The greatest common factor of 18 and 10 is 2. Thus, the answer is H.

 