Michael K. Smith, “Why is Pythagoras Following Me?” Phi Delta Kappan, February 1989, pp. 446-459. (Reproduced with the permission of Phi Delta Kappan magazine).

Remember the butterflies in your stomach on the morning you took a "college entrance" test? Remember your scores? More important, do you remember all the things you had to know in order to earn those scores? Could you be admitted to a college or university today? Before you continue reading this article, stop and try the sample test on the following page.

If you read right on to this paragraph without attempting the test on the next page, please be advised that your opinions on the reform of education may be suspect. If you did take the test, please return your answers to me, and your score will be mailed to you in six weeks.

Just kidding. The answers to the test questions appear later in this article. While reading on in search of them, perhaps you will notice some of the serious criticisms I am making about mathematics education and education reform in general. If this article seems a little mysterious so far, perhaps you will be better prepared to understand the feelings that many students experience in dealing with mathematics. For, in a sense, school mathematics is mysterious, and the current back-to-basics movement, which stresses the need of every student for more mathematics, seems only to heighten the mystery.

Let me be clear from the outset. The mathematics requirements in our nation's high schools should not be increased, as so many proponents of education reform proclaim. In fact, they should be decreased. Why so many people support the former proposition and so few consider the much more reasonable and logical alternative of the latter is the theme of this article. In particular, I will challenge the need for requiring between two and four years of abstract mathematics in our high schools. This requirement includes two years of algebra, one year of geometry, and one year of trigonometry and (perhaps) beginning calculus. I will focus on algebra and geometry because, whenever critics complain of declining scores on math tests, high school students generally end up taking more of these courses.

Sample Test

DIRECTIONS

Solve each of the problems in this section using any available space for scratchwork. Then decide which is the *best* of the choices given and circle the letter that corresponds to your choice.

Note: Figures which accompany problems in this test are intended to provide information useful in solving the problems. They are drawn as accurately as possible EXCEPT when it is stated in a specific problem that its figure is not drawn to scale. All figures lie in a plane unless otherwise indicated. All numbers used are real numbers.

Please begin work. You have five minutes.

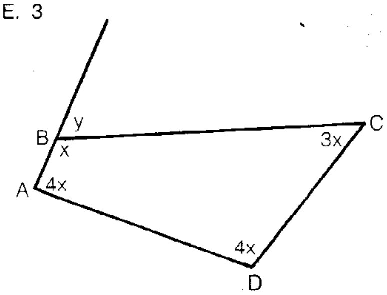
1. $(\sqrt{128})(\sqrt{32}) =$

   A. 64
   B. $32\sqrt{2}$
   C. $64\sqrt{2}$
   D. 128
   E. $64\sqrt{4}$

2. If $x^2 - 6x = -9$, then $x =$

   A. -3
   B. 0
   C. 1
   D. 2
   E. 3

3.

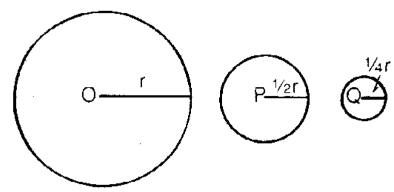


ABCD is a quadrilateral (not drawn to scale).

$y =$

   A. 30
   B. 60
   C. 90
   D. 150
   E. 180

4.



The area of circle O minus the area of circle P minus the area of circle Q equals

   A. $16\pi^2$
   B. $11\pi r^2$
   C. $11/16\pi r^2$
   D. $12/16\pi r^2$
   E. $15/16\pi r^2$

5. Sam drove part of a trip at 70 miles per hour, and Bill drove the other part at 50 miles per hour. If Sam drove for 2 hours and Bill drove for 1 hour, what was their average speed (in miles per hour) for the entire trip?

   A. 50
   B. 60
   C. $63\frac{1}{3}$
   D. $66\frac{2}{3}$
   E. 70

STOP

IF YOU FINISH BEFORE TIME IS CALLED, YOU MAY CHECK YOUR WORK ON THIS SECTION ONLY. DO NOT WORK ON ANY OTHER SECTION IN THE TEST.

DO NOT START READING THE REST OF THIS ARTICLE UNTIL THE FULL FIVE MINUTES HAVE ELAPSED.

Why teach mathematics at all? How can we justify to our children the learning of such material? The rationale for mathematics instruction rests on three virtually unchallenged assumptions. The first, bequeathed to us by Greek philosophers and supported by modern science, is that mathematics is the foundation on which the universe is structured. Mathematics thus becomes philosophy and offers a methodology that will reveal the elegant harmony that seems to underlie the physical and, perhaps, even the social world. The second assumption, which draws its strength from a naive brand of psychology, assumes that study of the logical, deductive, and inductive methods common to mathematics will enable one to think more clearly and more rationally about other subjects. This transfer-of-training argument hinges on the belief that understanding and practice in one area of knowledge will transfer to the learning of another. The third and more pragmatic premise is that mathematics is so frequently used in daily life that it should be taught to our children. The evidence supporting each assumption is far from clear. In fact, in many cases, the evidence supports exactly the opposite conclusions about mathematics. In the case of the first assumption, the correlation between simple numbers and the physical world - the Greeks notwithstanding - has not been substantiated. The field of mathematics itself has splintered into numerous disciplines, each postulating a different view of the relationship between mathematics and reality. Second, the psychological evidence for transfer of training not only does not support this assumption but confirms its opposite: that training in one area may actually interfere with the learning of another. Finally, no study has supported the contention that the abstractions of algebra, geometry, and trigonometry, which so many students are required to learn, are practical in any general sense, except for a small number of occupations.

When all three assumptions - the philosophical, the psychological, and the pragmatic - fail and students are still forced to learn more of something as abstract as high school mathematics, I claim the following: that the back-to-basics movement we are now encountering will not improve our children's mathematical abilities. Indeed, it may succeed only in creating a new generation that is either hostile to mathematics or anxious about dealing with it or both. How has this situation come about?

MATH AS PHILOSOPHY

To most adolescents in high school, math is algebra and geometry. In fact, we might argue that most adults, when asked an opinion about mathematics, will discourse on their experiences in one of these two subjects. Obviously, there are many different branches of mathematics. But, during the impressionable teenage years, the axioms, postulates, and logical rigors (or hazards) of these two particular areas tend to leave an indelible impression of the nature of mathematics. Why algebra and geometry? Why even teach mathematics? To answer these questions we must discuss the centuries-old fascination with the mysteries of mathematics.

In fact, we must come to know Pythagoras quite well. Since the days of the ancient Greeks, mathematics has been seen not only as useful and playful but also as possessing a certain kind of magic, a magic that might provide a way to unlock the mysteries of the universe. Plato's academy in Athens was reported to have had a sign above its entrance proclaiming, "Let no one enter here who does not know geometry." Plato echoed and extended what, even in his day, was beginning to be a commonplace idea: that number is the matter and form of the universe. The Pythagoreans rejoiced in the fact that "all things are number" and built a mystical cult around the worship of numbers.

Why such excitement? How could numbers that could be counted on your fingers or simple figures drawn in the sand initiate a philosophical approach to nature that would last more than 2,000 years?

The explanation lies partly in the properties of numbers and figures themselves and partly in their remarkable correspondence, as it seemed to the Greeks, to things in the real world. Every student in high school has probably had an encounter with the Pythagorean theorem, which states that, given a right triangle, the square of the hypotenuse is equal to the sum of the squares of the opposite two sides ($A^2 + B^2 = C^2$). The Pythagoreans proved that this principle would hold for all right triangles, thus discovering a pattern that should transcend all times and all spaces. No wonder that the Pythagoreans, when they found similar properties for squares, circles, and spheres, went on to proclaim that all nature is structured numerically.

These regularities probably would have remained only stimulating and interesting curiosities had it not been for the "discovery" that number could possibly describe the regularities of the physical universe. Applying numbers to the realm of astronomy, the Pythagoreans felt that the motions of planets and stars could be described mathematically. This connection between mathematics and the physical world inspired the first "standardized" school curriculum: the quadrivium, centered on four subjects linked by mathematics (arithmetic, geometry, music, and astronomy). Mathematical texts - particularly Euclid's *Elements*, the formalization of the propositions of geometry that is still at the heart of high school courses more than 20 centuries later - were revered and have been translated anew in each succeeding age. The mathematical method (and, by association, the philosophical and scientific methods), the deductive logic of geometry as codified by Euclid, cast a shadow over western civilization that is perhaps rivaled only by that of Aristotle or the Bible.

Abstractions and regularities had been seen to lie behind natural phenomena, and these nontransient facts became the focus of philosophy and science for the next several centuries. Galileo typifies the argument for a nature that was designed mathematically:

> Philosophy (nature) is written in that great book which even lies before our eyes - I mean the universe - but we cannot understand it if we do not first learn the language and grasp the symbols in which it's written. The hook is written in the mathematical language, and the symbols are triangles, circles, and other geometrical figures, without whose help it is impossible to comprehend a single word of it; without which one wanders in vain through a dark labyrinth.[1]

This naive view began to change in the 1800s, and the changes that occurred not only rocked the foundation of mathematics but also made a mockery of claims that nature, the universe, society, or anything else is structured so simply - or, indeed, is even describable by mathematics at all. The simple nature of algebra and geometry yielded to a more complex mathematics, and doubts were raised about the efficiency of any mathematical descriptions of the world. Morris Kline commented on these changes in *Mathematics: The Loss of Certainty*:

> The current predicament of mathematics is that there is not one but many mathematics and that for numerous reasons each fails to satisfy the members of the opposing school. It is now apparent

that the concept of a universally accepted, infallible body of reasoning - the majestic mathematics of l800 and the pride of man - is a grand illusion.[2]

How did this happen? In general, the decline has come about on two fronts: first, the strictly logical foundations of arithmetic, algebra, and geometry have been challenged to such an extent that there are now several competing algebras and geometries; second, the dream of describing the world with one unified mathematics has been seriously questioned as different mathematics have come to describe different realities. In short, the belief in a single objective reality, describable by a single logical and truthful mathematics, which is based on algebra and geometry, has simply not been supported.

During the late 19th century and into the 2Oth century, the challenges continued. The construction of matrix algebra showed that there were certain algebras in which the supposedly immutable law of commutation did not hold. In the 193Os Kurt Godel proved that no logical system complex enough to deal with numbers could ever be complete; that is, there may always exist statements that cannot be proved. Furthermore, the world, which was thought to be so rigorous, logical, and deterministic, was shown, at times, to be purely probabilistic. Probability theory came to be used more and more in areas ranging from particle physics to economics to psychology. And, finally, there was the computer. The logic and discrete mathematics of this tool proved to be different from simple algebra and geometry.

The foregoing historical digression is not meant to imply that the mathematics of algebra and geometry are inapplicable to anything in real life. Nor is it meant to suggest that any mathematical description is inappropriate. The efficiency of mathematics in describing certain natural phenomena and technological processes is unquestionable. I do mean to suggest, however, that the grand vision of a world structured according to geometry and algebra can no longer be accepted. As knowledge has progressed and as the world has changed, the study of mathematics has become more complex, and its ability to simply describe all phenomena has become more questionable.

But high school has not changed. The ancient Greek view of mathematics as algebra and geometry is still the core of mathematics in high schools. And even when alternatives are taught, their philosophical implications are seldom discussed. More calculation and more theory are given, but rarely is the human struggle with mathematics presented. Mathematics is mysterious - but only in the sense that it reflects the continuing human attempt to solve the mystery of the universe. Studying the history of mathematics and the various modern attempts to model the world mathematically would teach students valuable lessons: that knowledge has limits, that mysteries remain in the universe, and that humankind is engaged in a never-ending attempt to unravel those mysteries. Isn't this what students should learn from school if we wish to train them for future excellence?

MATH AND PSYCHOLOGY

The nature of the human mind has always been as mysterious as the fascination with mathematics. Perhaps this explains in part why the two have so often been linked. One of the most enduring notions that has been used to explain the mystery of the mind has been that the mind is composed of "faculties" or "powers" that, like the biceps, must be exercised to develop their strength. From the Greek philosophers to Descartes to the faculty psychologists and phrenologists of the 19th century to certain modern cognitive psychologists, the mind has been divided into elementary powers: reason, memory, language, logic, attention, perception, etc. To improve reason, one very important attribute of mind, one must exercise this faculty on the most difficult training equipment available: namely, mathematics.

For instance, John Locke wrote that:

> Would you have a man reason well, you must use him to it betimes, exercise his mind in observing the connection of ideas and following them in train. Nothing does this better than mathematics, which therefore I think should be taught all those who have the time and opportunity, not so much to make them mathematicians as to make them reasonable creatures.[3]

These philosophical speculations might remain merely academic exercises were it not for the profound practical implications of the belief in the primacy of mathematical thinking and of the assumption that ability or training in mathematics will transfer to other areas. For generations of college students, this belief has played an important role in determining their futures because a primary instrument used to judge admissions to college programs is the Scholastic Aptitude Test (SAT), which includes mathematics as *one-half* the total score. The only support for such a heavy emphasis on mathematics in this and other tests is the enduring belief that mathematical abilities are crucial for the success of all undergraduate and graduate students. Those who design and use the test (college admissions officers included) appear to believe that skill in mathematical reasoning should transfer to other areas that require logical thinking.

Such was the original belief of Carl Campbell Brigham. Working at Princeton from the mid-1920s to the late 1930s, Brigham developed several versions of the SAT. The one central purpose of his work was to assess a student's *general ability*, independent of specific cultural influences or peculiar high school training programs, in order to provide a test that would rank students from throughout the country. The test score came to consist of two major parts: verbal and mathematical. The SAT displayed good reliability (student scores did not seem to fluctuate much on retests) and fair validity (the test could reasonably predict college grades). From the verbal and mathematical abilities students demonstrated on such a standardized test, colleges and universities would be able to select students of outstanding ability and future promise. In at least this one context, the dictum of Plato that no one would succeed without geometry has proved to be true. But is this belief in the transfer of training justified?

The idea of a strong degree of transfer of training, based on mental faculties, runs deep in the mental psychologies and philosophies of the 19th century. However, no empirical evidence exists to support such a claim of transfer of training, especially the broad proposition that the learning of mathematics will facilitate, say, the learning of the logic of physics or of economics.

One of the earliest pieces of negative evidence comes from the celebrated William James. Using himself as subject, James tried to determine whether or not memorizing poetry would enhance his ability to memorize in general. First, he learned 158 lines of Victor Hugo's *Satyr* and kept a careful record of the time required. Next, he spent more than a month committing Milton's *Paradise Lost* to memory. Finally, he memorized another 158 lines of *Satyr*, which required more time than did the first 158 lines. James concluded that all his practice on *Paradise Lost* had not improved his "memory faculty"; in fact, he wasn't any better at memorizing poetry, the subject he had practiced on, much less at memorizing other, different material.

In an important paper published in 1901, Edward Thorndike and Robert Woodworth reported finding no evidence that improving one mental function would improve another. In their experimental tests of the effects on one another of such concepts as discrimination, attention, memory, and other mental functions, they found no evidence for a transfer of training. They concluded:

> Improvement in any single mental function rarely brings about equal improvement in any other function, no matter how similar, for the working of every mental function group is indicated by the nature of the data in each particular case.[4]

In a conclusion similar to James' disparaging conclusion about the effect of memorizing *Paradise Lost* on memory in general, Thorndike and Woodworth wrote that "improvement in any single mental function need not improve the ability in functions commonly called by the same name. It may injure it."[5]

In the 1920s Thorndike moved more directly into research on the learning of mathematics, especially the learning of arithmetic and algebra. In his 1923 volume on the *Psychology of Algebra*, Thorndike and his fellow researchers at Teachers College, Columbia University, criticized the techniques behind the teaching of the "old math":

> The faith in indiscriminate reasoning and drill was one aspect of the faith in general mental discipline, the value of mathematical thought for thought's sake and computation for computation's sake being itself so great that what you thought about and what you computed with were relatively unimportant.[6]

Drawing support from the belief in mental faculties, the "old math" had emphasized the value of mathematical reasoning in its own right, as a subject whose rudiments could easily transfer to the learning of other disciplines. Thus, algebra textbooks could deal with "bogus" and "fantastic problems" because the content wasn't nearly as important as the general principle behind the content.

It is well-known that scores on intelligence tests, including scores on the mathematics subtests of such standardized measures as the SAT, correlate well with academic grades. There is very little evidence, however, that they correlate with much else - particularly with those things that might predict success in later life. In a review of the relevant research, Michael Wallach notes that tests seldom predict real-life accomplishments. In a study of 500 undergraduates, Wallach found that original accomplishments outside the classroom - in literature, science, art, music, dramatics, political leadership, and social science -were unrelated to SAT scores, although high SAT scores were related to high grades. Citing a study by Lindsey Harmon, which interrelated the professional contributions of physicists and biologists with data on the academic proficiency of college students, Wallach wrote, "How good a professional scientist the person became could not be predicted from any of this academic proficiency information." Should we then have stronger test standards? Wallach concludes:

> The irony is compounded when educational commentators. . urge even heavier use of academic aptitude test scores instead of grades in selection on the ground that the former are less subject than the latter to irrelevant sources of bias such as teachers giving higher grades to students who are more polite. For it is their correlation with grades that provides the aptitude tests with their basic justification in the first place.[7]

Yet the belief in the power of mathematics as an aid to logical thinking continues. As a popularly held notion, we can perhaps countenance such a stand, for popular notions often reflect current fancy. As a professional opinion, however, we must be less tolerant of ill-founded notions. Educators and scholars should vigorously examine the validity of any idea that has such profound effects on the lives of children. But even among professionals, there is a wide difference of opinion concerning the power of mathematics.

In the early 1970s, Ohmer Milton of the Learning Research Center at the University of Tennessee surveyed assistant, associate, and full professors and members of the professional staffs on several college campuses. He asked the respondents to agree, disagree, or say that they were undecided about the truth of the statement: “the study of mathematics is especially important in helping students learn to think logically." Sixty-two percent of all professors in natural, biological, and physical sciences agreed with the statement; only 18% disagreed.

The trend among faculty members in other disciplines and among members of professional staffs was exactly the opposite. Only 26% of professors in the humanities agreed; only 32% of those in the social sciences agreed; only 40% of members of professional staffs agreed. The only truly strong support for the generalizability of training in math comes from the "hard" sciences - those that have longer histories of using mathematics as models for research. For other university people, the belief that mathematics is a tool that disciplines the mind is not widespread. As Milton concluded, "The fact that so many of the 'hard' scientists believe in such a fundamental issue in the absence of the kinds of data they themselves would accept may be presumptive evidence against automatic subject matter transfer."[8]

The notion that the study of mathematics will help in the study of other subjects is simply erroneous. There is no experimental evidence to support the broad idea of transfer of training. Given that math is not the key to the universe or the key to learning other material, if math is still to be universally taught, then only one argument remains: math must be very practical and useful. As Thorndike hoped,

> emphasizing ability to use algebra in solving problems which life will offer, it seems desirable to consider the lives of boys and girls and men and women as students, citizens, fathers and mothers, lawyers, doctors, business men, or nurses, and select problems which they may usefully solve and which are properly solved by algebraic methods.[9]

MATH AND PRACTICALITY

Is the math that is now being taught in the nation's high schools useful in the lives of the ordinary citizens of this nation, as Thorndike hoped it would be? If we can demonstrate such practicality for the mathematical topics that are normally covered in high school, then an argument for retaining these courses can be made. If not, we must consider why math is taught at all, and what effect the teaching of something so impractical has on the students who are forced to take it.

In the following discussion, remember that I am talking about the high school mathematics quadrivium - algebra I and II, geometry, and either trigonometry or precalculus. Let me not deal in abstractions: those who continue to suggest that "math skills" are declining and that "math standards" must be raised should bear in mind how these suggestions are translated in the textbooks used in high school classes.

Even in the 1920s, Thorndike complained of the "fantastic" and "bogus" problems that were used in high school algebra textbooks. These problems were usually organized by the algebraic technique involved and not according to their use in real life. If anyone were to scan the textbooks used in these courses today, I think that he or she would find that the situation hasn't changed much from the turn of the century.

For example, let's investigate the teaching of algebra. In a detailed analysis of topics in algebra, Thorndike felt that only a few could be called basic principles: the idea of symbolism; the ability to read formulas; the ability to evaluate and solve formulas, first in one unknown and then in two or more; and, finally, the ability to read graphs. How many of us who have had two years of algebra could tell someone else so succinctly what the essentials of algebra are?

It should not be surprising if we failed. For from these simple notions, students are drilled in countless variations on the basic themes. The ability to read formulas, for example, might come in handy, if we are asked to evaluate interest rates, to convert degrees Fahrenheit to degrees Celsius, to convert miles to kilometers, or, perhaps, even to solve a physics problem. However, endless pages of drill are devoted *not* to such practical matters but to the evaluation of such equations as:

Divide y^3-$6y^2$+ 14-12
by y-3

Or multiply $2a^2$ - a - 28 / ($3a^2$ -9- 2)

by $3a^2$+11a+6 4/(a^2+ 16a+7)

Homework often consists of solving endless variations on a simple principle. Those who feel that I am exaggerating should talk to their children.

Most textbooks do attempt to provide some practical applications of these concepts through word problems. All of us who teach any kind of postsecondary mathematics know the shudders that run through our classes when "word problems" are mentioned. Why? A brief examination of any algebra textbook will reveal that these attempts at practicality often become exercises in absurdity. Consider these examples:

Sam can do a job in five hours working by himself. Bill, however, can do it in three hours. How long will it take them if they work together?

* * *

Miguel is eight years older than his brother, Juan. Five years from now, he will be twice as old as Juan. How old are the boys now?

* * *

Jonathan can bicycle from his home to school in 12 minutes while his older sister Suzanne can make the same trip in eight minutes. If Jonathan starts from home to school at the same time that Suzanne starts from school to home, how long will it be before they meet?

Although all three examples have "algebraic solutions," how sensible are the assumptions that we must make to find the correct answer? When two people work together on a task, simple psychology indicates that the time to complete it depends on many factors. Similarly, when two people start out on a trip, a time when they will meet is contingent on several things. For anyone who has lived a normal life, these last two statements are only common sense. Finally, concerning the age problem, how many people would not know the ages of their own brothers?

The fact that most word problems seem to involve purely mathematical techniques or tricks would be fine if they didn't masquerade as real-life, applied concepts of algebra. The silliness of these exercises fosters a sense of absurdity in students. The techniques they are learning are only nonsensically applied, so why should they bother to learn the material? As Thorndike suggested, solving problems in school ought to be for the sake of solving problems in life. Thus problems should be organized to match techniques with real-life applications.

Now we might ask, Just how much of the high school mathematics quadrivium can be used in everyday situations? Once again Thorndike is a good guide. In his 1923 book he examined a sample of articles from many sources - encyclopedias, almanacs, and so on - to see how much algebra was actually incorporated into real-world settings. His conclusions were mostly negative: only a fraction of the high school algebra that was then taught was actually used. Is this surprising? From our own experience, how often do we use the math that we learned in high school?

I think these techniques are used only under certain limited conditions in certain jobs - mostly engineering and other technically oriented occupations. Must all students endure four years of math for the sake of producing a few technicians?

In one critical respect, the answer to this question is a definite yes, for nearly all high school students who plan to attend college are forced to deal with math in a way that affects their futures. I am referring once again to college admissions tests. In most cases, the math required on these tests is high school algebra and geometry.

Could you be admitted to college today? If you didn't take the test at the be-ginning of this article, now is a good time to do so. If you did, the answers are as follows: 1. A; 2. E; 3. D; 4. C; 5. C. To determine your admissions status, use the following college admissions key (approximate, of course): one correct (no admission); two correct (admission with remedial math work); three correct (general admission); four correct (admission with a good chance at a scholarship); five correct (admission with a great chance at a scholarship or fellowship).

Given this approach, students see math as nonsensical material that they are coerced into learning, not as a freely chosen, inherently interesting subject. And coercion has its own deterrent effects, as Einstein observed:

> One had to cram all this stuff in-to one's mind for the examinations, whether one liked it or not. This coercion had such a deterring effect on me that, after I passed the final examination, I found the consideration of any scientific problems distasteful to me for an entire year.[10]

MATH ANXIETY

So far in this discussion, I have tried to shatter the assumptions that support educators' reliance on the teaching of mathematics: that mathematical models reflect, in a simple way, the structure of reality; that training in mathematics will facilitate training in other subjects; and that mathematics, as defined by high school courses, is useful in the workaday world. If all of these are unfounded assumptions, what justification remains for requiring - and perhaps even increasing the requirements for - these subjects? More important, what effects might we expect these increased requirements to have on high school students, forced to deal even more intimately with mathematics?

Reasonable criteria for increasing the amount of mathematics available to students in high schools do exist. We read constantly of our nation's need for technical expertise, for more scientists, engineers, and computer scientists to help keep the country strong in research and development and a continued world leader in technology. To this end, sophisticated and abstract courses in science and mathematics must be offered in both high school and college.

However, since only a small minority of high school students actually enter these professions or even wish to do so, must all high school students be forced to endure a highly specialized mathematics curriculum in order to select the few who are needed? Must all students, whatever their future plans, be forced to deal with two to four years of such highly abstract technical subjects as algebra and geometry, which will be useful only in a relatively small number of professions?

Such is indeed what happens and will continue to happen at an accelerated pace if the current proposals for revising high school curricula are adopted. An even greater number of high school students will have to endure even more algebra and geometry in order to graduate. Those who are more inclined toward the liberal arts and who see no future at all in mathematics must either learn the subject or accept the reduced chances of admission to and scholarships from major universities that are the direct result of lower high school grade-point averages and lower scores on college admissions tests.

For those who encounter math, endure it, but don't like it, we might expect "math anxiety" to develop, a fear that might keep them away from any math-oriented disciplines. Recent research on math anxiety has shown the phenomenon to be real and to have an impact on people's perceptions of themselves and of their career choices.

In *Overcoming Math Anxiety*, Sheila Tobias describes the sexual biases that are built into mathematics curricula. Males are more highly reinforced for their mathematical abilities; the problems in textbooks are slanted toward a male perspective (males get the rewarding mathematical tasks: Johnny does math on the job, Susie in the kitchen); and even the behavior of teachers shows evidence of bias toward males. Thus increasing the number of required mathematics courses not only favors future technicians and scientists, but also probably favors males who have traditionally chosen to pursue mathematically oriented careers in greater numbers than females. Obviously, this trend might change as more females enter these professions. But how many never get the opportunity to do so because of some early discouragement concerning their mathematical ability?

Certainly, not everyone will come to hate math. However, repeated exposure to math in high school, reinforced by the impracticality of the material, can only plant questions in the minds of even the most capable students: For what purpose? Is there a reason for all this, other than the mental gymnastics involved? If no answers to these questions emerge from students' later lives, then high school courses facilitate a "cult" of mathematics. That is, the belief grows that the material must have some value, somewhere, sometime. The student doesn't know where or when, but surely somebody must know. An air of mystery then surrounds mathematics, a numerical magic that keeps us from questioning the users of numbers, for we feel that what they do must be right, for we believe that mathematics is always right!

In forcing students to take such courses we are creating a generation of students more hostile to and more anxious about mathematics than preceding generations. And for the vast majority of them, the mystery and magic of numbers grows deeper.

WHAT TO DO?

I have tried to show that the rationale for the way we present mathematics in high school is implausible. We cannot continue to force our children to take ever-heavier loads in such subjects as algebra, geometry, and trigonometry, courses that even adults cannot justify in terms of practical significance. What, then, should be the place of mathematics in high school and beyond?

1. *The history and philosophy of mathematics should be taught to all students.* To eradicate the mystery that needlessly attaches itself to numbers, we must acknowledge their human attributes; that is, we must explain how and why people have used mathematics to understand their world. Perhaps we should call this a "math appreciation" course. In such a course, we could try to justify the study of math, rather than simply force everyone to study years of algebra and geometry.

2. *The only math requirement in high school should be one year of basic math.* This course

would include practice in simple arithmetic skills, as well as training in the use of calculators and in such simple applications of mathematics as interest rates (as used by banks, savings and loan associations, department stores, etc.), mortgages and loans, and tax calculations. The use of real-life equations can be introduced, and students can be taught the essential elements needed to solve them.

Furthermore, I think it is crucial that a section on "government math" be included. That is, students should receive an introduction to the ways in which large organizations (industry could also be included) arrive at various estimates, from gas mileage to defense spending. One of the major goals of schooling in a democratic society, as Jefferson foresaw, was the production of educated citizens. I can't think of anything more practical than lessons in how real-life mathematics works. Students need the opportunity to learn real-life problem solving that involves the use of mathematics - not simply rote drill.

3. *Algebra I, algebra II, geometry, and trigonometry/precalculus should become electives.* No student should be required to take these courses. Those who are inclined toward these subjects should certainly have the opportunity to study them in depth. Others who couldn't care less would be better served by more time in literature or history courses.

Some might claim that, by eliminating the requirement that all students take these math courses, we will risk overlooking a host of talented young people who might become researchers in technology and science. I claim just the opposite: more of these professionals would be produced, as long as the demand for them exists. High school students who show an early interest in mathematics and science will pursue these subjects as electives, and we needn't be concerned with these groups of students. From those who develop these interests later, say, in the first or second year of college, we might indeed recruit more engineers and technically oriented professionals. However, bad experiences in high school math can steer people away from mathematically oriented professions and prevent these late bloomers from following up on their interests. Eliminate the bad experiences, and we might well have more people choosing technical professions at a more mature age. Most universities could teach the prerequisite math necessary for beginning engineering students in a one- or two-semester sequence. If students in these courses want to be there, the material could probably be taught even faster.

When I teach statistics and research design to undergraduate students in psychology, I have often wished that my students had had no high school mathematics. What they remember of the content of these courses is usually minimal. But they bring with them a crippling desire not to take any "math-related" course again. Pythagoras haunts them and affects how I must now teach them.

4. *The math sections of the major standardized tests, such as the SAT, should be eliminated.* Math is not that important to many professions, and to require all students to master it is ludicrous. By doing so, we only deny admission to some otherwise capable students and make people in the tutoring business rich.

The central issue is one of choice and freedom. We automatically assume that the cure for "declining standards" is more of everything: more hours of schooling, more homework, more math courses, and so on. In solving educational problems, we seldom stop to think about learning. We seldom ask how we are going to instill in our students the desire to learn and to create instead of just the ability to make the grade.

High schools are currently designed to foster confinement, not to encourage freedom. They consider constraint, not freedom, to be the essence of American education. In this way, our high schools hope to produce a new generation of talented individuals to invent new technologies, to discover new ideas, and to create new products to keep the country strong. But coercion does not encourage curiosity, as Einstein noted:

> It is, in fact, nothing short of a miracle that the modern methods of instruction have not yet entirely strangled the holy curiosity of inquiry; for this delicate little plant, aside from stimulation, stands mostly in need of freedom; with-out this it goes to wreck and ruin without fail. It is a very grave mistake to think that the enjoyment of seeing and searching can be promoted by means of coercion and a sense of duty.[11]

I believe that we cannot go forward to the future by going "back" to the basics. Freedom of choice is what has always made this country great. Isn't it time we put it back into our classrooms and gave it back to our children?

REFERENCES

1. Quoted in Morris Kline, *Mathematics: The Loss of Certainty* (Oxford: Oxford University Press, 1980), p.46.

2. Ibid., p.6.

3. Thomas Fowler, ed., *Locke's Conduct of Understanding* (New York: Macmillan, 1980), p.20.

4. Quoted in William S. Sahakian, *Learning: Systems, Models, and Theories* (Chicago: RandMcNally, 1976), p.100.

5. Ibid.

6. Edward L. Thorndike, *The Psychology of Algebra* (New York: Macmillan, 1923), p.96.

7. Michael A. Wallach, “Tests Tell Us Little About Talent," *American Scientist*, vol.64, 1976, p.63.

8. Ohmer Milton, Learning Transfer: A Diogenian Task," *Teaching-Learning Issues*, vol. 15, 1971, p.8.

9. Thorndike, p.109.

10. Dean Keith Simonton, *Genius, Creativity, and Leadership* (Cambridge, Mass.: Harvard University Press, 1984), p.63.

11. Ibid.